Attachment E — Correspondence with the Lean Formalization of the OA Construction

Lauri Korpela
MiulusTek Oy, Finland · lauri.korpela@miulustek.com

Attachment E to Structural Classification of the Exterior-Exponent Collapse Mechanism (Paper I), 2026. Examined 15 September 2026.

Section 8 of Paper I compares the structural class with an external construction, OpenAI's *Finite Time Blowup for Navier–Stokes* [2], called OA below. Paper I treats OA as a realization only conditionally on its correctness. OpenAI has also released a Lean 4 formalization of the construction [9]. This attachment shows where each OA-dependent statement of Paper I appears in that formalization, so that readers can compare the conditional statements with Lean declarations.

Paper I's own results do not use the formalization, and the status labels in Paper I and its claim ledger are unchanged by what follows. Bracketed numbers refer to the reference list of Paper I.

1. The formalization examined

Item	Value
Repository	`openai/NavierStokesAndEuler` [9], Apache-2.0
Commit	`f9e8bc5b38b6e212696e8a30e3e91517af887bbd` (10 September 2026)
Toolchain	Lean `v4.34.0-rc2`, Mathlib `85e3a25`, Comparator `19e111e`
Main theorems	`NavierStokes.Comparator.navier_stokes_breakdown_R3` and `NavierStokes.Comparator.navier_stokes_breakdown_periodic`, stating the Clay alternatives (C) and (D). The project aligns them with Theorem 1.1 and Corollary 10.6 of [2] and describes its own review status as "self-assessed".
Permitted axioms	`propext`, `Quot.sound`, `Classical.choice`

The correspondence below comes from reading the Lean source at this commit. The proof modules contain no `sorry`, `admit` or additional `axiom` declarations. The `sorry` placeholders of the Comparator challenge file are not imported by the proof. We did not compile the project or run its axiom and Comparator checks. The independent audit [10] reports that it could not complete those steps either.

2. Theorem numbering

Comments in the Lean source cite an earlier numbering of the OA manuscript. For example, the cone criterion is cited as "Lemma 3.5", the leading stress as "Proposition 3.2" and the terminal blow-up argument as "Proposition 11.7". In the public version of [2], the cone criterion is Lemma 4.5, and there are no numbered results in a Section 11. This attachment cites [2] by its public numbering and matches Lean declarations to it by content.

3. The main results of OA in Lean

Result in [2]	Lean declaration	Content
Theorem 3.1: a local construction with constants $0 < h < 1/100$, $q_* > 0$ and $0 < X_a < X_{\text{ext}}$	`LocalPaper.local_theorem`, with its clauses in `LocalPaper.Properties`	Smooth fields with a curl decomposition, zero divergence, flat residual, an exact heat exterior and angular growth
Theorem 1.1, whole space	`NavierStokesR3.theorem_1_1`, leading to `Comparator.navier_stokes_breakdown_R3`	Clay alternative (C)
Corollary 10.6, periodic case	`PeriodicPaper.periodic_corollary`, leading to `Comparator.navier_stokes_breakdown_periodic`	Clay alternative (D)
Existence of the constructed flow	`ActualCandidateAssembly.selected_candidate`	Proves the proposition `ProblemStatement.candidateStatement`

4. Paper I statements and their Lean counterparts

The last column uses four terms. **Stated**: a Lean theorem states the property. **Partial**: the property appears in definitions or supporting lemmas, but no single theorem states it in the form used by Paper I. **Not found**: no matching declaration was found. **Not formalized**: outside the scope of [9].

	Paper I statement	Source in [2]	Lean declarations	Correspondence
L1	§8: the parameter satisfies $0 < h < 1/100$	Theorems 3.1 and 4.6	`LocalPaper.Properties.exponent_small`; `LocalAngularGrowth.selected_exponent_small`	**Stated** for the exponent chosen in the construction
L2	§8: exterior circulation $\Gamma_{\text{ext}} \sim r^{-2h}$, so $\alpha = 2h$	Theorem 3.1 (heat exterior); Lemma 4.9; Appendix A	`BaseExterior.heatVelocity`; `LocalHeatFormula`, where the swirl amplitude is r^{-1-2h} times an exterior profile of τ/r^2; `LocalPaper.Properties.exterior`	**Stated** for the swirl amplitude; the circulation ru^θ follows on multiplying by r

	Paper I statement	Source in [2]	Lean declarations	Correspondence
L3	§8: core lengths $\rho \sim s^{1/2}$, $\ell \sim s^{1/2-h}$ and growth exponent $a_{\text{grow}} = h$	Section 2	`Scaling.radialLength`, `Scaling.axialLength`, `Scaling.coreVelocity`; `LocalPaper.AngularRayGrowth`	Lengths **partial** (defined scalings). Growth **stated**: the swirl velocity behaves like $\tau^{-(1/2+h)}$ at a fixed similarity radius, which gives $a_{\text{grow}} = h$
L4	§8: core energy $s^{1/2-3h}$	Computed in Paper I	—	**Not found**. The energy results in [9] are global bounds, as in `NavierStokesR3.theorem_1_1_with_dissipation`
L5	§8: exterior cone parameter $v_s - 2 = \alpha$	Lemma 4.5 and (4.20); Lemma 4.9 and (A.56)	`ConeAlgebra.true_cone_iff`; `SeedConePaper.admissible_iff`, with `shearMagnitude a b` $= a(1 + (b/a)^2)$ equal to v_s in (4.20); `UniformCone.compact_trueCone_margins`	Cone criterion **stated**; the exterior value **not found**. On its outer collar [2] gives $2 + h < v_s \leq 2 + 2h$ rather than an equality. Paper I's identity concerns its own exterior profile (Attachment B)
L6	Premise A1b and §8: no interaction stress in the core	Theorem 4.6(ii)	`FinalSlowBase.stressForce_core_germ`; `FinalSlowBase.stressForce_jets_zero_in_exterior`; `AssembledSlowBase.coefficients_stress_zero_left`	**Stated** for the base flow: the stress vanishes inside and outside the active annulus. Not stated separately for the complete interaction field
L7	§8 and ledger N03: moment repair	Section 5; Appendix A	`MomentRepair`; `FiveProfileMoments`; `ParameterizedMomentRepair.exists_smooth_uniform_branch`; `GlobalStressSupport`	**Partial**. The formalization repairs five moments; Paper I's two-moment problem (Level A) has no separate counterpart

	Paper I statement	Source in [2]	Lean declarations	Correspondence
L8	§8 and ledger N04: oscillatory realization with a uniform cone gap	Appendix C; Proposition 7.5	`TrueConeLoopPaper.exists_local_family_with_margin`; `ModulatedCone`; `UniformCone.compact_family_trueCone_stable`	**Partial**. `ModulatedCone` addresses whether radial modulation preserves the cone gap, the question raised in [10], §4.6
L9	Ledger C93: OA realizes the class	Theorems 1.1 and 3.1	`ActualCandidateAssembly.selected_candidate`; `LocalPaper.local_theorem`; the main theorems of Section 3	**Stated**
L10	Paper I §§2–7: supply theorem, the window $0 < \alpha < 1/3$, cone identities, midplane, fold models, Theorem 7.1	—	—	**Not formalized**

5. Scope

The table locates statements. It does not show that the Lean definitions encode the corresponding notions of [2] or of Paper I, and it does not certify the proofs of [2]. Two steps would turn the correspondence into a verification: compiling [9] at the commit above and checking the axioms of its main theorems, and comparing each declaration listed in Section 4 with the text it is matched to. The search was by name, docstring and statement, so declarations under unrelated names may have been missed; “not found” should be read in that sense.