Attachment B — Cone Realizability and the Midplane Construction for Lemma H

Stress realizability for the exterior-exponent architecture

Lauri Korpela
MiulusTek Oy, Finland · lauri.korpela@miulustek.com

Attachment B to Structural Classification of the Exterior-Exponent Collapse Mechanism (Paper I), 2026. Companion to Attachment A, §9.

> **How to read this attachment.** This note develops the cone half of Lemma H and follows the midplane construction of Attachment A, §9. It presents the analysis in the order it was carried out; statements that later sections corrected are kept and marked as withdrawn or superseded, and the claim ledger accompanying Paper I gives the current status of every claim. "OA" denotes OpenAI's construction [2], cited by its own equation and theorem numbers. Theorem numbers of the form 9.x refer to Attachment A. Bracketed citations refer to the reference list of Paper I.

---

0. Result, stated first

The cone half of Lemma H is **not an obstruction**. It is solved exactly and unconditionally.

> **Theorem C1 (the cone, in closed form) [EXACT].** With OA's normalised radial shear $s = (a, -b_s)$, $a > 0$, and integrated inviscid-stress vector p_s, the demanded stress is $T_0 = F(p_s - s)$, $F > 0$, and the admissible stress cone is the **circular** cone
>
> $$\mathcal{C} = \left\{ T \in \mathbb{R}^2 \colon\ T \cdot s > 0,\ \ \angle(T, s) < \theta_c \right\}, \qquad \theta_c = \arctan \sqrt{\frac{2a}{|s|^2 - 2a}},$$
>
> non-degenerate precisely when $|s - e_1| > 1$, $e_1 = (1, 0)$. Its **axis is the shear direction** s.

> **Theorem C2 (pointwise realizability) [EXACT].** Given p_s, an admissible shear exists **iff**
>
> $$p_{s,2} \neq 0 \quad \text{or} \quad p_{s,1} > 2.$$
>
> The cone condition is therefore infeasible only on the closed ray $\{p_{s,2} = 0,\ p_{s,1} \leq 2\}$, a set of measure zero.

> **Theorem C3 (on-axis realizability) [EXACT].** A shear placing the demand exactly on the cone axis ($J_c = 0$, maximal aperture margin) exists **iff**
>
> $$\Theta := |p_s|^2 - 2p_{s,1} = |p_s - e_1|^2 - 1 > 0,$$
>
> and then v_s may be chosen anywhere in $\left(2,\ 2 + \Theta / p_{s,1}\right)$.

So both s and p_s are governed by the **same** unit circle centred at $(1, 0)$: s must lie outside it for the cone to exist, and p_s outside it for the demand to be placeable on the axis.

What is **not** yet solved is a profile admissible in height. §9C shows why that is a design question rather than a trajectory question. §9E constructs the active midplane layer (Attachment A, Theorems 9.13–9.15). Attachment A Theorem 9.18 shows that the reflection-symmetric midplane cannot be completed inboard of the layer, and Attachment A §9.16.9 completes the non-symmetric one: the inboard region is made stress-free by a linear lift (Theorem 9.25) that is uniquely solvable above an explicit amplitude threshold (Theorem 9.26). The midplane is therefore closed. Continuing it in height is neither an initial-value problem

nor a march from the stagnation surface. The stress-free equations are parabolic in η, with direction set by the axial velocity (Attachment A, Theorem 9.27). The surface where that velocity vanishes carries no intrinsic trace and cannot be horizontal inboard (Attachment A, Proposition 9.28 and Theorem 9.29), and both ends of the height interval are outflow. An intrinsic clock does not split it: each sign of the axial velocity is forward-parabolic along its own drift, but the height slices along which the fields diffuse cross the stagnation set at finite radial distance, so inboard the two sides are one system (Attachment A, Proposition 9.30 and Theorem 9.31). The inboard region is a single forward–backward degenerate parabolic boundary-value problem. Its singular set cannot be designed down to one point: a fold at the axis and a flat junction at the handover corner are forced, and sinks appear in every tested lift (Attachment A, Theorem 9.33). The energy route to existence is closed, although the equations are not. No bounded positive local diagonal multiplier symmetrizes the coupled height equations (Attachment A, Theorem 9.34). The operator multiplying the height derivative is the midplane non-resonance operator on every height slice, and it is indefinite with infinite index on the slice through the forced axis fold (Attachment A, Theorem 9.35). At that fold the equations survive every test so far. The scalar fold is well posed and smoothing (Attachment A, Theorem 9.36). The swirl–pressure pair turns the fold core into a quasi-elliptic problem of type $(2, 6)$, with boundary conditions inherited from the original variables (Attachment A, Theorem 9.37). Core and outer fold are the same two modes, with fixed eigenvectors (Attachment A, Theorem 9.38). Their junction reduces to one scale-invariant normal form (Attachment A, Theorem 9.39). That normal form is the classical Kolmogorov forward–backward equation in a nonlocal gauge, uniformly well posed with no data at the zero (Attachment A, Theorem 9.40). At the axis the swirl–pressure pair becomes self-adjoint in the natural degenerate weights, and one energy covers the whole fold with no height data (Attachment A, Theorem 9.41). What is left is a single small skew shear term (Attachment A, Proposition 9.42). No energy multiplier can absorb it approximately, because a degenerate mass demands an exact symmetrizer (Attachment A, Lemma 9.43). The operator route replaces the multiplier: the fold is uniformly invertible at sufficiently small height frequency (Attachment A, Proposition 9.44), and at large height frequency the local models have trivial kernels on their specified domains, with global exhaustion now supplied by Attachment A Theorem 9.60. At the axis the two modes decouple exactly and the mass has no kernel (Attachment A, Theorem 9.47 and Lemma 9.48). Attachment A Lemma 9.50 now proves a uniform whole-line normal-form lower bound through zero Fourier frequency: the weighted Green operator fails on arbitrary forcing through range compatibility, which does not produce quasimodes. The global argument proves that every hypothetical quasimode concentrates at the axis (Proposition 9.51). The pressure endpoint supplies an inviscid reflection relation (Proposition 9.52), and the proposed transport model is uniformly invertible with that relation (Lemma 9.53). Proposition 9.54 transfers the pressure relation to actual viscous primitive coefficients on a shrinking outer collar, and Proposition 9.55 gives local compactness and identifies the square-integrable homogeneous transport limits. Proposition 9.56 passes the outer reflection to those limits and proves they vanish. Proposition 9.57 transfers this vanishing to physical norm at or above the transport scale. Theorem 9.60 now closes full frozen-window high-frequency compactness: Proposition 9.58 controls every fixed $|\tau|^{-1/6}$ core, and Lemma 9.59 controls the intervening six-component system with justified endpoint states. No normalized high-frequency quasimode remains. §10 states the verdict.

---

1. Lemma H, precisely stated

Fix $\alpha \in (0, \frac{1}{3})$ and a matching interval $I = [X_a, X_b] \Subset (0, \infty)$ in the similarity radius.

Data. A profile $\Phi = (E, U, \Pi)$ on I, in $C^k(I \times [-1, 1])$ for the k the construction requires, regular at the axis in the Cartesian sense, joined to a prescribed exterior with circulation exponent α.

Derived maps. From Φ:

$$\Phi \;\mapsto\; R=(R_\theta,R_z) \;\mapsto\; T=(T_{r\theta},T_{rz}), \qquad T_{r\theta}=-\frac{1}{r^2}\int_0^r \sigma^2 R_\theta\,d\sigma, \quad T_{rz}=-\frac{1}{r}\int_0^r \sigma R_z\,d\sigma,$$

$$\mathcal{M}[\Phi,\alpha] := \Big(\int_0^\infty r^2 R_\theta\,dr,\ \int_0^\infty r R_z\,dr\Big)\in\mathbb{R}^2, \qquad \mathcal{C}[\Phi,\alpha](r)\subset\mathbb{R}^2 \text{ as in Theorem C1.}$$

The feasibility problem.

$$\boxed{\mathcal{M}[\Phi,\alpha]=0 \quad\wedge\quad T[\Phi,\alpha](r)\in \operatorname{int}\mathcal{C}[\Phi,\alpha](r)\ \ \forall r\in I,\ \text{with a margin uniform on } I.}$$

$$\mathcal{A} := \big\{\alpha\in(0,\tfrac{1}{3}):\ \exists\,\Phi_\alpha \text{ solving the above}\big\}.$$

Level A / Level B. The above is **Level A**, the reduced problem Paper I needs: two moments, a two-component cone, one shared profile. OA's Theorem 4.6 is **Level B** and carries more — axis regularity, an exact heat exterior, **five** cumulative moments, pressure matching, edge weights and two reserved intervals. Nothing here proves Level B, and Level A is not a weakened restatement of it: Level B's five moments include Level A's two.

---

2. The cone, computed

2.1 Identification [EXACT, verified symbolically]

From OA (4.11), (4.20), (4.23): $t_s=-b_s/a$, $v_s=a(1+t_s^2)$, $P_c=p_{s,1}+t_s p_{s,2}$, $J_c=p_{s,2}-t_s p_{s,1}$, and $T_0=F(p_s-s)$ with $s=(a,-b_s)$. Writing $w:=p_s-s=T_0/F$ and $s^\perp=(b_s,a)$, all verified in `sympy`:

$$v_s=\frac{|s|^2}{a}, \qquad (1,t_s)=\frac{s}{a}, \qquad P_c-v_s=\frac{w\cdot s}{a}, \qquad J_c=\frac{w\cdot s^\perp}{a}.$$

Hence OA's (4.23), $X:=T_{0,\theta}+t_sT_{0,z}=F(P_c-v_s)$ and $Y:=T_{0,z}-t_sT_{0,\theta}=FJ_c$, hold identically.

2.2 The cone is circular, with axis s [EXACT]

The map $(T_\theta,T_z)\mapsto(X,Y)$ has matrix $\begin{pmatrix}1 & t_s\\ -t_s & 1\end{pmatrix}$, which satisfies $M^\top M=(1+t_s^2)\mathbb{I}$ — **conformal**, a rotation by $-\arctan t_s$ with scaling. Angles are therefore preserved, and OA's admissibility

$$P_c>v_s, \qquad (v_s-2)J_c^2<2(P_c-v_s)^2$$

becomes $w\cdot s>0$ and $\tan^2\angle(w,s)<2a/(|s|^2-2a)$, which is Theorem C1. Three readings:

- **the cone axis is the radial shear**, so steering the shear steers the cone;
- $v_s>2\Leftrightarrow|s|^2>2a\Leftrightarrow|s-e_1|>1$ — the shear must lie **outside the unit circle centred at** $(1,0)$;
- $v_s\to 2^+$ opens the cone to a half-plane; $v_s\to\infty$ closes it onto the axis; $a\to 0$ also closes it.

2.3 Audit of Lemma 4.5, against Gondauri §4.3 [EXACT + NUM]

Gondauri flags that "squaring and thereby losing sign information could artificially enlarge the cone". The concern is **real in principle and is not a defect in OA**.

The polynomial $2(P_c-v)^2-(v-2)J_c^2$ has roots $v_\pm=P_c+\frac{J_c^2}{4}\pm|J_c|\sqrt{\frac{P_c-2}{2}+\frac{J_c^2}{16}}$ — verified symbolically — and $\mathcal{U}(P_c,J_c)=v_-$, so (4.21) is $P_c>2$, $v_s<v_-$. The squared inequality alone is invariant under $P_c-v_s\mapsto-(P_c-v_s)$ and so admits the **opposite half-plane** $T_0\cdot s<0$: on 1.9×10^5 random admissible-v_s samples, **65.9%** of points satisfying the squared inequality violate $P_c>v_s$. OA states $P_c>v_s$ alongside it in (4.22), which restores exactly the lost sign. **[NUM]** the equivalence (4.21)$\wedge(v_s>2)\Leftrightarrow$(4.22) was tested on 3.9×10^5 samples with $v_s>2$: **0 mismatches**.

> **Verdict on the Gondauri flag: no defect.** The sign is carried explicitly. It is one of the few places where the audit names a concrete algebraic risk.

---

3. Pointwise realizability, solved

3.1 Reduction [EXACT]

Write $s = r(\cos f, \sin f)$ with $a = r\cos f > 0$, so $f \in (-\frac{\pi}{2}, \frac{\pi}{2})$; $p_s = |p_s|(\cos\psi, \sin\psi)$; $\vartheta = \psi - f$. Then admissibility is

$$\text{(i) } r > 2\cos f, \qquad \text{(ii) } r < |p_s|\cos\vartheta, \qquad \text{(iii) } (r - 2\cos f)|p_s|^2\sin^2\vartheta < 2\cos f\,(|p_s|\cos\vartheta - r)^2.$$

At $r = 2\cos f + \varepsilon$ the left side of (iii) is $O(\varepsilon)$ while the right tends to $2\cos f\,(|p_s|\cos\vartheta - 2\cos f)^2 > 0$ whenever (i)–(ii) leave room. **So (iii) is not binding near the lower endpoint**, and

$$\text{feasible} \Leftrightarrow \exists f \in (-\tfrac{\pi}{2}, \tfrac{\pi}{2}):\ 2\cos f < |p_s|\cos(\psi - f) \Leftrightarrow \sup_{\cos f > 0} \left[(p_{s,1} - 2)\cos f + p_{s,2}\sin f\right] > 0.$$

On the open right half-circle the supremum is $|p_{s,2}|$ if $p_{s,2} \neq 0$ (let $f \to \pm\frac{\pi}{2}$), and $(p_{s,1} - 2)^+$ otherwise. **Theorem C2.**

[NUM] certification. An explicit witness s was constructed for every predicted-feasible case and exhaustively excluded for every predicted-infeasible one, on 14 test points including $(1.9, 0.3)$, $(0.9, 0.2)$, $(-1, 0.5)$, $(0.05, 0.05)$ and $(1, 1)$: agreement on all. A first, coarse grid search had reported three of these as infeasible; the witnesses require $\varepsilon = r - 2\cos f$ as small as 10^{-7} with $a \sim 10^{-2}$, which the grid could not resolve. **The analytic reduction, not the search, is the result.**

3.2 On-axis realizability [EXACT]

$J_c = 0 \Leftrightarrow w \parallel s \Leftrightarrow p_s \parallel s$. Substituting $s = \lambda p_s/|p_s|$ into (i)–(ii) gives $2\cos\psi < \lambda < |p_s|$, feasible iff $|p_s|^2 > 2p_{s,1}$, i.e. $\Theta > 0$; and then $v_s = \lambda/\cos\psi$ sweeps $\left(2,\ |p_s|^2/p_{s,1}\right)$. **Theorem C3**, and the endpoint $2 + \Theta/p_{s,1}$ matches numerical suprema over the parallel family to discretisation error (2.487 vs 2.5; 3.312 vs 3.333; 2.269 vs 2.273).

3.3 What is not true [FAIL, with counterexample]

It is tempting to conclude that $\Theta \leq 0$ forces $v_s \to 2^+$, so that $\Theta > 0$ is the uniform-margin criterion. **That is false.** At $p_s = (0.5, 0.2)$, where $\Theta = -0.71$, a search over non-parallel shears attains $v_s \approx 2.219$. Off-axis shears buy v_s-margin at the cost of aperture margin, and no closed form for the resulting two-parameter trade-off was obtained here.

> **Consequently the correct statement of the cone half is C2 and C3, and not a single uniform-margin criterion.** The conjecture that $\Theta > 0$ is necessary for a uniform margin is **withdrawn**; only its sufficiency for the *on-axis* margin is proved.

---

4. Proof attempt (Approach A): local openness

Claim. Let α_0 admit Φ_0 with $\mathcal{M}[\Phi_0, \alpha_0] = 0$ and $\operatorname{dist}\left(T(r), \partial\mathcal{C}(r)\right) \geq \delta_0 > 0$ uniformly on I. Then $(\alpha_0 - \varepsilon, \alpha_0 + \varepsilon) \cap (0, \frac{1}{3}) \subset \mathcal{A}$.

Ingredients required, and their status.

ingredient	status
$\mathcal{C}$ is **open**, so a strict margin persists under small perturbation of T and of s	**[EXACT]**, Theorem C1 — the inequalities are strict and the data enter continuously
$(\Phi, \alpha) \mapsto \mathcal{M}$ is C^1 in the bump coefficients	**[COND-OA]** — OA's corrections are finite linear combinations of smooth compactly supported bumps, with changes to the moments "linear or quadratic in the coefficients" (§4.2)
the moment Jacobian is invertible on the correction directions	**[COND-OA]** — asserted by OA Lemmas A.1, A.2; **not verified here**, and Gondauri records that the full existence lemmas have not been independently reproved
the moment repair does not close the cone gap	**[COND-OA]** — OA's $N\log X$ modulation has amplitude $O(N^{-1})$ with order-one change in radial derivatives; Gondauri's §4.6 [10] states the correct test: if the repair's C^2 norm is $O(N^{-1})$ and the gap is fixed and positive, one finite N suffices

Status: the implicit-function argument goes through given the three [COND-OA] ingredients, and only given them. Two of the three are OA lemmas this note has not reproved. So Outcome A is obtained **conditionally**, and must not be read as a theorem of Paper I.

Why this is nevertheless worth stating. The structure of the argument is now visible, and the separation that makes it work is explicit: the modulation moves the **shear** s at order one while moving the **integrated** quantity p_s, and hence $T_0 = F(p_s - s)$, at $O(N^{-1})$. Since the cone axis *is* s (Theorem C1), this is precisely a mechanism for steering the cone onto a nearly fixed demand. That is the design principle of OA Appendix C, stated in one line.

5. Obstruction test (Approach C), independently run

Four candidate obstructions were tested. **None succeeded.**

candidate	outcome
the demanded direction crosses $\partial\mathcal{C}$ as α grows	**not established.** The cone axis is a design variable (Theorem C1); a crossing would have to survive optimisation over s, and by C2 that is possible only on a measure-zero ray
one component of T scales faster than the other as α grows	**false.** Both residuals are $O(q^{-A-1})$ and both $T_{r\theta} \sim \rho R_\theta$ and $T_{rz} \sim \rho R_z$ are $O(q^{-1-h})$ — the same order, matching OA's own $A^2_{\text{wave}} \asymp q^{-1-h}$. **There is no scaling obstruction; the question is $O(1)$ geometry**
compact support forces the direction of T to rotate out of any convex cone	**false.** The cone inequalities are homogeneous, so $T \to 0$ at an edge is compatible with a fixed limiting direction; $T_{r\theta} = -P/r^2$ with P vanishing at both ends may be one-signed, and the moments constrain only P's endpoint values, not its sign

candidate	outcome
moments and cone conflict through a sign or shape restriction	**not established at Level A.** The two moments fix two scalars; the cone constrains a direction pointwise. No incompatibility was found

> **The obstruction test fails, and informatively: the cone half of Lemma H obstructs nothing.** If Lemma H is false, it fails on the **joint** problem — moments and cone for the same Φ — or at Level B, not on the cone alone. That is exactly the point Gondauri identifies as critical, and this note's contribution is to remove one of the two halves from suspicion.

---

6. Where the proof stops, exactly

Everything above is a statement about p_s. Nothing here computes $p_s(r;\alpha)$.

p_s is the normalised radial primitive of the inviscid tangential residuals (OA (4.9), (4.16), (4.11)), so obtaining $\Theta(r;\alpha) = |p_s|^2 - 2p_{s,1}$ requires integrating the inner profile system with the exterior matching — the content of OA Theorem 4.6 and Appendices A–B. **This note did not do that, and without it the α-dependence of realizability is inaccessible.**

Concretely, the first missing object is:

$$\Theta(r;\alpha) \text{ on } I, \text{ and whether } \inf_{r\in I}\Theta > 0 \text{ persists as } \alpha \uparrow \tfrac{1}{3}.$$

Everything in §§2–3 is then a decision procedure applied to it.

---

7. Continuation alternative (Approach B)

What can be asserted about $\mathcal{A}$, given Theorems C1–C3:

> **Either realizability continues in α, or one of the following occurs**, and the list is now explicit rather than schematic:
>
> 1. $\det D_\Phi \mathcal{M} \to 0$ — the moment repair loses rank;
> 2. $\inf_r \Theta(r;\alpha) \to 0$ — the on-axis margin of C3 closes;
> 3. $a(r) \to 0$ somewhere on I — the cone collapses onto its axis even though C2 still holds;
> 4. $p_s(r)$ approaches the ray $\{p_{s,2} = 0,\ p_{s,1} \le 2\}$ — the only pointwise infeasible set, by C2;
> 5. the edge limits of $T/|T|$ leave the closed cone — the "edge weights / common vanishing factors" condition;
> 6. loss of axis regularity, support separation, or exterior matching.

Items 2–4 are new and are checkable from the profile alone. $\mathcal{A} \neq \varnothing$ **is [COND-OA]** via the seed $\alpha_0 = 2h < \frac{1}{50}$; no upper bound $\alpha_c < \frac{1}{3}$ was found, and none is claimed.

---

8. OA comparison

required at Level A	present in OA?	location	status
two moment conditions	yes, as two of **five**	§3.2; Lemma A.8; Prop. B.8	**[COND-OA]**
two-component demanded stress $T_0 = F(p_s - s)$	yes	Table 1; (4.11)	**[EXACT]** as read
cone as positive span of two covariance directions	yes	§3.2; Prop. 7.5; Fig. 4	**[COND-OA]**
cone in closed form	**derived here**, not stated there	§2.2 above	**[EXACT]**
strict inclusion	yes, and OA notes homogeneity permits $T \to 0$ at edges	(4.23); Thm 4.6(iii)	**[COND-OA]**
quantitative lower bound on the gap	asserted; Gondauri: pointwise gap $\neq$ uniform gap on a noncompact domain	§A.5	**[OPEN]**
free parameters after the moments are fixed	the $N\log X$ modulation: $O(N^{-1})$ values, $O(1)$ shear	§3.2 step 4; Prop. C.2–C.3	**[COND-OA]**
deformations preserving moments	separated bumps restoring all five	Prop. B.8; Lemmas A.1–A.2	**[COND-OA]**
deformations moving the cone	the same modulation, since the axis **is** the shear	**derived here**	**[EXACT]**

9A. p_s computed from the profile [EXACT]

§6 said the missing object was $p_s(r;\alpha)$. It is now computed where the profile is explicit, and the answer identifies OA's cone parameter with Paper I's exterior exponent.

9A.1 The dictionary [EXACT]

OA (4.11) sets $a = 1 - 2D_X \log E = 2 - 2l$ with $l = D_X \log H$, $H = \sqrt{2X}\,E$ the angular-momentum profile, $D_X = X\partial_X$. Since $X \propto r^2$ at fixed η, $D_X = \frac{1}{2} r\partial_r$; and $\Gamma = r u_\theta^{(0)} = q^{-h} H$, so

$$l = \frac{1}{2}\frac{d\log\Gamma}{d\log r} = -\frac{1}{2}\alpha_{\mathrm{loc}}(r), \qquad \alpha_{\mathrm{loc}}(r) := -\frac{d\log\Gamma}{d\log r},$$

and therefore

$$\boxed{a = 2 + \alpha_{\mathrm{loc}}(r), \qquad b_s = \frac{2D_X U}{E} \text{ (normalised axial shear).}}$$

OA's radial-shear parameter is the local circulation exponent, shifted by two. That is the bridge between the two papers, and it is an identity, not a mapping.

9A.2 The exterior, in closed form [EXACT]

OA's exterior has $U = V_0 = 0$ and $E = c_\infty X^{-1/2-h}$, whence $H = \sqrt{2}\,c_\infty X^{-h}$, $l = -h$, and with $\alpha = 2h$:

$$s = (2+\alpha,\ 0), \qquad v_s = \frac{|s|^2}{a} = a = 2+\alpha, \qquad \boxed{v_s - 2 = \alpha}, \qquad \theta_c = \arctan\sqrt{2/\alpha}.$$

The pressure follows from $\Pi_X = E^2/2X$ normalised at radial infinity: $\Pi = -\frac{c_\infty^2}{2(1+\alpha)} X^{-1-\alpha}$, so $D_X\Pi = -(1+\alpha)\Pi$, and the axial source of (4.9) collapses:

$$S_n = 4A\eta\Pi + 2\eta D_X\Pi = \eta\Pi\big[\,2(1+\alpha) - 2(1+\alpha)\,\big] \equiv 0,$$

the cancellation being exact because $4A = 2(1+\alpha)$. Hence $D_X N_s + N_s = 0$ in the exterior, so XN_s is constant there: precisely the r^{-1} tail OA removes.

> **The moment condition and the stress are the same object.** "$\int_0^\infty r R_z\,dr = 0$" is exactly "$XN_s \to 0$", i.e. the vanishing of that constant. The two halves of Lemma H were never independent conditions to be checked separately and combined — the moments *are* the statement that the stress returns to zero, and with them $p_s = s$ and $T_0 = F(p_s - s) = 0$ in the exterior, consistent with OA Proposition 4.2.

9A.3 The cone in profile variables [EXACT, verified symbolically]

$$v_s - 2 = \frac{\alpha_{\rm loc}(2+\alpha_{\rm loc}) + b_s^2}{2+\alpha_{\rm loc}}, \qquad \theta_c = \arctan\sqrt{\frac{2(2+\alpha_{\rm loc})}{\alpha_{\rm loc}(2+\alpha_{\rm loc}) + b_s^2}},$$

and at either annulus edge, where $T_0 = 0$ and so $p_s = s$,

$$\Theta = |p_s|^2 - 2p_{s,1} = |s|^2 - 2a = \alpha_{\rm loc}(2+\alpha_{\rm loc}) + b_s^2.$$

> **Theorem C4.** $v_s > 2 \Leftrightarrow \alpha_{\rm loc}(2+\alpha_{\rm loc}) + b_s^2 > 0$, for which $\alpha_{\rm loc} > 0$ is sufficient. Where the axial shear vanishes it is also necessary. Hence
>
> **OA's cone-admissibility condition $v_s > 2$ is Paper I's $\alpha > 0$.**
>
> In the exterior the two are literally the same inequality. **The condition for the ambient circulation to rise inward — which is what makes the swirl grow — is the condition for the wave family to be able to supply the stress at all.** They were derived independently, from a kinematic argument here and from a covariance constraint there, and they coincide.

Where $\alpha_{\rm loc} < 0$ the axial shear can still rescue admissibility, via $b_s^2 > |\alpha_{\rm loc}|(2+\alpha_{\rm loc})$. With $b_s = 0$ and $a > 0$ it cannot: $\Theta > 0$ then forces $l < 0$, so **the matching annulus must lie outside the angular-momentum maximum** — which is exactly the centrifugally unstable region where OA amplifies its pulses. The cone algebra and the amplification mechanism require the same geometry.

9A.4 α-dependence: the cone improves as α grows [EXACT]

α	$v_s - 2$	θ_c	Θ
0.002	0.002	88.2°	0.004
0.02	0.02	84.3°	0.040
0.1	0.1	77.4°	0.210
0.2	0.2	72.5°	0.440
1/3	1/3	67.8°	0.778

> **There is no cone degeneration at the top of the window.** As $\alpha \to \frac{1}{3}$ the admissibility margin reaches $\frac{1}{3}$ and the aperture is still 67.8°. The cone degenerates at the **other** end: $\alpha \to 0$ sends $v_s - 2 \to 0$, and that is where OA operates ($\alpha < \frac{1}{50}$, margin < 0.02). The suspected obstruction at $\alpha = \frac{1}{3}$ is refuted for a second time, now by computation rather than by absence of evidence.

9A.5 Uniformity, and Gondauri's compactness objection [EXACT]

Gondauri (§4.3, §A.5) objects that a pointwise strict inequality need not give a uniform gap on a noncompact domain. The objection separates cleanly here.

- **On the annulus** $I = [X_a, X_b] \Subset (0, \infty)$, which is **compact**, α_{loc} and b_s are continuous, so pointwise positivity of $\alpha_{\text{loc}}(2 + \alpha_{\text{loc}}) + b_s^2$ **does** give a uniform gap. No further argument is needed.
- **On the noncompact exterior tail**, where his objection would bite, the computation of §9A.2 gives $v_s - 2 = \alpha$ **identically in** X **and** η — a constant, not merely a positive function. And there $T_0 = 0$, so the angular part of the cone condition is vacuous.

> **The uniform-margin problem is therefore structurally closed.** It reduces to pointwise positivity on a compact interval, and the tail is uniform by exact computation. What is *not* closed is the pointwise condition in the **interior** of the annulus, which needs Q_s, N_s there and hence the profile.

9A.6 Restoring the moments without closing the gap

The margin is a strict inequality in quantities that the moment repair perturbs. OA's repair adds separated bumps of size $O(N^{-1})$; Gondauri states the correct test, and it is met in form: **if the repair is** $O(N^{-1})$ **in the norms controlling** a, b_s **and** p_s, **and the gap is fixed and positive, one finite** N **suffices.** The delicate point is that $a = 2 + \alpha_{\text{loc}}$ involves a *radial derivative*, so a repair small in C^0 is not automatically small in a; it must be small in C^1, and the $N\log X$ modulation is deliberately **not** — that is how it steers the cone. The two must therefore be kept in separate radial regions, which is exactly OA's construction. **[COND-OA]**, not verified here.

9B. What is now proved, and what is left

statement	status
$a = 2 + \alpha_{\text{loc}}$, $b_s = 2D_X U/E$	**[EXACT]**
exterior: $s = (2 + \alpha, 0)$, $v_s - 2 = \alpha$, $\theta_c = \arctan\sqrt{2/\alpha}$	**[EXACT]**
$S_n \equiv 0$ in the exterior; the axial moment condition is $XN_s \to 0$	**[EXACT]**
$v_s > 2 \Leftrightarrow \alpha_{\text{loc}}(2 + \alpha_{\text{loc}}) + b_s^2 > 0$; $\alpha_{\text{loc}} > 0$ suffices	**[EXACT]**
OA's $v_s > 2$ **= Paper I's** $\alpha > 0$	**[EXACT]** in the exterior; sufficient in general
the annulus lies outside the angular-momentum maximum (when $b_s = 0$)	**[EXACT]**
no cone degeneration as $\alpha \to \frac{1}{3}$; degeneration is at $\alpha \to 0$	**[EXACT]**
uniform margin: compactness on I; constant margin on the tail	**[EXACT]**
Θ in the **interior** of the annulus	**[OPEN]** — needs Q_s, N_s, i.e. the inner profile

statement	status
moment repair preserves the gap	**[COND-OA]**

Lemma H″ : do admissible profiles (E, U) exist with $M_0, M_1, M_2 > 0$ on the annulus?

> **Obsolete formulations, withdrawn.** An earlier ending of this note reduced the survivor to $\inf_r \Theta > 0$. That is **only the on-axis criterion** (Theorem C3) and is sufficient, not necessary; §3.3's counterexample already showed off-axis shears do better. And the two-part form above is itself superseded by §9C: the margins are M_0, M_1, M_2, and the problem is a design problem rather than a pointwise inequality along a trajectory.

Both halves are now conditions on the **same two profile functions**, α_{loc} and b_s, on a compact interval. That is as far as this note goes without integrating the inner system.

9C. The inner profile: Outcome G [EXACT]

A natural route is to ask whether $\mathcal{R} = \{M_0, M_1, M_2 > 0\}$ is forward-invariant under the inner-profile ODE, and to prove realizability by an invariant region. **There is no such ODE.** That is the result.

9C.1 The exact stress system [EXACT, verified symbolically]

With $w_1 = p_{s,1} - a$ and $w_2 = p_{s,2} + b_s$, OA's (4.9)–(4.11) are **equivalent** to

$$\boxed{D_X w_1 + l\, w_1 = \Sigma_q, \qquad D_X\left(E w_2\right) = \Sigma_n,}$$

$$\Sigma_q := \frac{X S_q}{L} - D_X a - l a, \qquad \Sigma_n := \frac{X S_n}{L} + 2 D_X^2 U.$$

Σ_q, Σ_n are the **zero-stress defects**: they vanish identically iff the profile solves the vanishing-residual equations, and OA's Proposition 4.2 is the statement that $w = 0$ then.

9C.2 Why this is not an invariant-region problem

(i) Not closed. S_q contains $H_c(\log E)_\eta$ and S_n contains U_η, Π_η; both contain W, which carries $\partial_\eta A_X(U)$. These are **axial** derivatives. No finite-dimensional radial state $Y = (\alpha_{\text{loc}}, b_s, p_{s,1}, p_{s,2})$, or any finite extension of it, reconstructs them: the realizability system is a PDE in (X, η).

> **Outcome G, proved.** The missing state is the **axial-derivative family** $(\partial_\eta \log E,\ \partial_\eta U,\ \partial_\eta \Pi,\ \partial_\eta A_X(U))$, and it is independent of the radial state by construction — it is precisely the information that a radial trajectory discards. Attempting to close the system in Y alone discards exactly this information.

(ii) The profile is designed, not evolved. Q_s, N_s are *determined* by (E, U, Π) via (4.10) with $C_Q = C_N = 0$ selected by axis regularity. So $M_i = M_i[E, U](X, \eta; \alpha)$ are functionals of free profiles.

(iii) Hence $\mathcal{R}$ is neither forward-invariant nor forward-repelling. A linear equation with free forcing is steerable to either side of any margin. The attempted proof route of Approach A is therefore not merely difficult but **ill-posed**, and it is also unnecessary: nothing drags the margins anywhere.

9C.3 Edge factorisation [EXACT, at simple zeros]

At an edge X_e with $w(X_e) = 0$, the system gives $w_1 \sim \Sigma_q \log(X/X_e)$ and $w_2 \sim (\Sigma_n/E)\log(X/X_e)$, so

$$\widehat{w}(X_e) = \left(\Sigma_q,\ \Sigma_n/E\right)\big|_{X_e},$$

and the edge condition is $\widehat{w}$ in the **closed** cone. **The edge datum is the zero-stress defect.** This supplies the directional information Approach B required, and it does so without needing OA's weight factorisation: the leading factor is $\log(X/X_e)$ and it cancels from the direction.

9C.4 The obstruction test, run again [FAIL]

Against six candidate obstructions: a forced crossing of $M_2 = 0$ cannot be established, because nothing forces the trajectory; $\alpha_{\text{loc}} < 0$ too large for b_s to rescue M_0 is possible pointwise but is a statement about a chosen profile, not a consequence; and incompatibility between the moments and a one-signed stress direction was already refuted in §5. **No obstruction found.**

9C.5 What the third unification would have to be, and why it was not found

Approach C asked whether M_1 or M_2 reduces to an existing profile condition, as M_0 reduced to $\alpha_{\text{loc}} > 0$. It does not, and the reason is structural: M_0 depends only on s, which is **local** in the profile (a and b_s are radial log-derivatives), whereas M_1, M_2 depend on $w = p_s - s$, and p_s is a **radial primitive** of sources containing axial derivatives. A local condition cannot be equivalent to a nonlocal one. **The first unification succeeded because M_0 is local; the remaining margins are not, and no further merge is available at this level.**

9D. The tangent problem, and the elimination of the cone [NUM + EXACT]

Outcome G says the demanded stress obeys a *driven linear* system whose sources carry axial derivatives no radial state records. That rules out the trajectory approach, and it also says what to do instead: **fix the background and linearise.** Once s_* is frozen at order ϵ^0, cone membership stops being a nonlinear coupling between the demand and the supply and becomes a condition on the direction of a single linear image.

9D.1 The linearised residual map [EXACT, validated]

On the midplane the state is (E, v, U_η) with $v = U(\cdot, 0)$, $U_\eta = \partial_\eta U(\cdot, 0)$, and $S_q = -Wl - h$, $S_n = -WD_X v - Av - vU_\eta$, $W = 1 - A_X(U_\eta)$. With $\psi = \delta E/E$,

$$\delta S_q = -WD_X\psi + l\,A_X(\delta U_\eta), \qquad \delta S_n = -WD_X\delta v - (A + U_\eta)\delta v - v\,\delta U_\eta + A_X(\delta U_\eta)D_X v.$$

δw then solves the same driven system as w, with integrating factors H and E. Two independent checks: a centred difference of the nonlinear residual map agrees to 1.6×10^{-8} relative at $\epsilon = 10^{-4}$, and the assembled coefficient matrix reproduces $\mathcal{L}$ to 2.8×10^{-13} (superposition, hence linearity, confirmed).

9D.2 Feasibility [NUM, 64/64]

$T \in \mathcal{C}(s)$ is equivalent to $T \cdot s > 0$ and $|T \cdot s^{\perp}| < \tan\theta_c\,(T \cdot s)$: two **linear** inequalities. With the two moment conditions also linear, feasibility is a linear programme over Φ_1, and it is feasible — in all 64 admissible backgrounds tried ($\alpha \in [0.02, 0.32]$, four layers in $[8, 55]$, two dip amplitudes), with Φ_1 supported strictly inside the layer, δw strictly inside $\mathcal{C}(s_*)$ at **every** grid point of I, angular margin ≥ 0.18 rad, and both moments preserved to 10^{-10}–10^{-13}. Attachment A Theorem 9.9.

The consequence is Attachment A Corollary 9.11, and it is **[COND]**: *given* one admissible profile whose residual vanishes on a layer carrying $M_0 > 0$, **and given that the tangent feasibility observed here holds there with a strict uniform margin**, Lemma H'' follows. The cone is discharged pointwise (§3, **[EXACT]**) and tangentially across a tested family (§9D.2, **[NUM]**); the analytic statement is §9E.

9D.3 Two facts the programme reports back [NUM]

The axial-flux profile must move, independently. $\delta U_\eta \equiv 0$: infeasible in all 12 backgrounds tested. δU_η tied to δv: infeasible. $\delta U_\eta \equiv 0$ with the moment constraints *dropped*: still infeasible. So the obstruction to the reduced parameterisations is the cone, not the moment bookkeeping — Outcome G restated as a condition on perturbations.

The axis is unreachable in the tested non-zero-stress family [NUM, that family only]. $|\delta w \cdot s^{\perp}| \le \kappa(\delta w \cdot s)$ is feasible at $\kappa = 0.20 \tan\theta_c$ and infeasible at 0.15. The reachable directions form a proper subcone: the covariance constraint is satisfiable but **not saturable**, and there is no optimal tangent perturbation. This supersedes an earlier calculation in this note that placed δw exactly on the axis with $J_c = 0$; that imposed neither the moments nor the requirement that δw be the image of an actual perturbation, and it is withdrawn. The statement is not a property of the architecture: at the zero-stress layer base of §9E the residual lies exactly on the axis, $w = \varepsilon(\chi, 0)$ (Attachment A, Theorem 9.13).

9D.4 What it is not

The test backgrounds are not zero-stress: $|w_*| \in [1.15, 2.58]$ on I, with $w_* \cdot s_* < 0$ throughout. The theorem is about δw, not $w_* + \epsilon\,\delta w$, and Corollary 9.11 is conditional for that reason. No near-solution is exhibited. What is exhibited is that the question remaining at this stage carries no cone condition. *(Superseded: §9E constructs the zero-stress base, and Attachment A §9.16.9 closes the midplane.)*

9E. The zero-stress base, and Lemma H on the midplane [EXACT + NUM]

The layer equation. $w_1 \equiv 0$ is $\Sigma_q \equiv 0$, which with $a = 2 - 2l$, $S_q = -Wl - h$ is

$$2X\,l' = 2l(1 - l) + XWl + Xh.$$

At $l = 0$ this gives $l' = h/2 > 0$ regardless of W or v. Since $l(0^+) = 1$, **a globally zero-stress profile has $l > 0$ and hence $\alpha_{\mathrm{loc}} < 0$ everywhere.** The layer inherits its exponent; it cannot make one. The equation degenerates at $h = 0$, where the cone degenerates (§9A.4).

The flux constant. Demanding $\alpha_{\mathrm{loc}} \equiv \alpha$ on the layer forces $\int_0^X U_\eta\,dx = 2 + \alpha$ there — identical to $c_U'(0) = 2 + \alpha$ from the exterior axial balance, reached by a different route. The flux is carried inboard; $l \equiv -h$ with $W = 1 - (2 + \alpha)/X$ solves the layer equation exactly.

The base. Take $l \equiv -h$, $v \equiv 0$, $U_\eta \equiv 0$ on I, flux $2 + \alpha$ inboard. The one remaining condition, $w_1(X_a) = 0$, is **not** a shooting problem: integrating by parts gives the closed functional

$$w_1(X_a)H(X_a) = -X_a H(X_a) + (1 - h)\int_0^{X_a} H\,dX - \int_0^{X_a} U_\eta H\,dX,$$

and along $l_\lambda = l_0 - \lambda\beta$ with $\mathrm{supp} U_\eta$ **outboard** of $\mathrm{supp}\beta$, the U_η term is λ-independent, leaving $F'(\lambda) = (1 - h)\int (B_\infty - B)R_\lambda\,dX > 0$. With $F(-\infty) < 0 < F(+\infty)$ the zero is **unique**. Attachment A Theorem 9.14; the numerical shooting is withdrawn.

The perturbation is finite, not tangent. A_X is linear and $S_q = -Wl - h$ is linear in W, so with $\psi = \delta v = 0$ and $\delta U_\eta = -(D_X\chi - h\chi)'/h$ the map is *exactly* affine in ε: $w = \varepsilon(\chi, 0)$ for every ε, with $\delta c_Q = \int (H\chi)' = 0$ and $\delta c_N = 0$ identically, and E, H, l, a, b_s untouched. Since $b_s \equiv 0$, $s_* = (2 + \alpha, 0)$ and w is a positive multiple of the axis: $J_c = 0$, margin $\theta_c = \arctan\sqrt{2/\alpha}$, no smallness condition anywhere.

The edges are forced. Beyond $\mathrm{supp}\chi$ the source vanishes, so $w_1 H$ is constant and $c_Q = w_1 H|_{\mathrm{out}}$; $c_Q = 0$ therefore forces χ to vanish at both edges. Strict cone membership is a statement about the *open active layer* $\{\chi > 0\}$.

Status [EXACT | TRANSCRIPTION]. Every step is algebra on the transcribed source equations; no numerical step survives. **Lemma H holds on the active layer of the fibre $\eta = 0$, analytically** — a closure of the layer, not of the fibre. With $U(\cdot, 0) \equiv 0$ the axis-regular preparation region necessarily carries demand where no cone exists (Attachment A, Theorem 9.18), so reflection symmetry is excluded for the global problem and a midplane axial velocity is needed inboard. Off the midplane, an earlier wedge $|\eta| < c\varepsilon\chi(X)$ and the conclusion that the layer endpoints must move are **withdrawn** (Attachment A, §9.16.8). The first-order axial moment forces $\int E_0\, \partial_\eta^2 E|_0 > 0$; truncations of the axial-flux-driven continuation return cusps they cannot resolve; and circulation-driven edge zones give fixed edges on full rectangles, conditional on four matching identities in η. The non-symmetric inboard completion is closed: with $\partial_\eta E|_0 \neq 0$ the inboard region is stress-free by a linear lift, uniquely solvable above an explicit amplitude threshold (Attachment A, Theorems 9.25–9.26), and the Return Lemma is redundant.

Three corrections. The "$l < -h$ on the layer" requirement assumed $W \equiv 1$; with the flux restored the condition is that $Wl + h$ change sign, and the undershoot moves inboard. The reading that $u_z(r, 0) \neq 0$ is necessary fails *on the layer* — $v \equiv 0$ here — though the actual necessary condition, at least three sign changes of $(w_1 H)'$, is met (four, for this design). For the global midplane, $v \not\equiv 0$ inboard *is* necessary, by a different proof (Attachment A, Theorem 9.18). And §9D.3's unreachable axis is a property of non-zero-stress backgrounds only.

10. Verdict

Outcome: the cone is solved, the active midplane layer is constructed, and the global midplane is closed [EXACT | TRANSCRIPTION]. The earlier outcome — reduction to one near-solution — is superseded: the near-solution was built on the layer, the obstruction moved inboard (Attachment A, Theorem 9.18), and Attachment A §9.16.9(h) removes it. With $\partial_\eta E|_0 \neq 0$ the inboard preparation is stress-free by a linear lift (Theorem 9.25). An energy identity with a Hardy bound proves that lift non-resonant and uniquely solvable whenever $\min_{[0,X_0]} v^2 > \int_0^{X_0} E_0^2\, l/x$ (Theorem 9.26); below that threshold resonances do occur, at a discrete, transversal sequence of amplitudes. The Return Lemma is redundant. In height, the closed midplane is not a Cauchy surface: the stress-free equations march in the direction of the axial velocity, the formal η-hierarchy closes to every order but diverges, and continuation from the midplane fails on the backward side (Attachment A, Theorem 9.27). Neither marching from the stagnation surface nor an intrinsic-clock split survives: the stagnation set is no design manifold (Attachment A, Theorem 9.29), and inboard the two signs of the axial velocity form one system joined along the height slices (Attachment A, Theorem 9.31). Its singular geometry is partly forced — an axis fold and a flat corner — and partly design-dependent, with sinks generic in the tested lifts (Attachment A, Proposition 9.32 and Theorem 9.33). The compatibility question for (NR) and (NR_η) is superseded. The scalar (NR_η) is not a coercivity condition of the coupled system, and no bounded positive local diagonal multiplier repairs that (Attachment A, Theorem 9.34). The height mass operator coincides with the Theorem 9.26 operator at the midplane and has infinite negative index on the axis-fold slice (Attachment A, Theorem 9.35). **That index is not a PDE obstruction.** The scalar axis fold is Hadamard well posed and hypoelliptic (Attachment A, Theorem 9.36). At the fold point the swirl–pressure pair sets a quasi-elliptic $(2, 6)$ core. Its time traces are transmission traces rather than data, and its three radial conditions are inherited from the u-, pressure- and e-traces and are complementing (Attachment A, Theorem 9.37). The transition to the outer fold involves no change of eigenvectors (Attachment A, Theorem 9.38). Its mode zeros reduce to a scale-invariant normal form with uniformly bounded connection constants (Attachment A, Theorem 9.39). The normal form is exactly the Kolmogorov equation $xb_\tau = \nu b_{xx}$ after the Fourier conjugation $a = |D|^c b$. It is uniquely solvable with one incoming trace per end, no data at the zero and an energy identity with constant 1, uniformly in frequency and Richardson number (Attachment A, Theorem 9.40). Every transmission obstruction test fails. At the

axis the swirl–pressure pair is self-adjoint in the weights $\nu E^2/l$ and ν, which Theorem 9.34 excluded only across the crossing. So H_c plus the pair carries one energy through core, corner and outer fold. The symmetric fold is well posed on small windows with no height data and no resonance (Attachment A, Theorem 9.41), and the corner is no longer a separate problem. The glued-symmetrizer route is [FAIL / REDUNDANT] (Attachment A, Lemma 9.43). Quasimode exclusion replaces it. The local models have trivial kernels on their specified domains, and Attachment A Theorem 9.60 now supplies global exhaustion. At the axis the swirl–pressure pair and the diffusion decouple **exactly** into two modes through the intertwiner $\frac{1}{r}\partial_r$ (Attachment A, Theorem 9.47). The fold mass has no kernel, its near-kernel being the mode-zero curve (Attachment A, Lemma 9.48). Quasimodes of bounded energy are impossible (Attachment A, Proposition 9.49). **Attachment A Lemma 9.50 proves the missing whole-line normal-form lower bound**, including zero Fourier frequency: $\| P_{\nu,\sigma,c} a \|_2 \geq \nu^{1/3} |\sigma|^{2/3} \| a \|_2$, uniformly for Re$c = -1/2$. Finite weighted graph norm forces the Fourier trace to vanish at zero; the unrestricted Airy Green operator fails a range compatibility, not this estimate. **The global argument now proves axis concentration for every hypothetical quasimode** (Proposition 9.51). A simple cutoff-and-freezing estimate has incompatible width requirements at the comparable-frequency scale. The proposed global transport limit must instead retain the pressure-induced reflection relation (Proposition 9.52); Lemma 9.53 proves its uniform invertibility with that relation, while omitting it admits a kernel. Proposition 9.54 now transfers the pressure relation through the outer viscous layer to a shrinking collar with vanishing normalized error. Proposition 9.55 makes the corrected primitive amplitudes precompact on fixed rescaled annuli and identifies their square-integrable homogeneous transport limits. Proposition 9.56 passes the pressure reflection to these limits and forces them to vanish. Proposition 9.57 uses an exact differential reconstruction to remove physical mass outside every fixed fraction of $\rho_{|\tau|}$. **Theorem 9.60 closes full (C*) and the stated large-frequency target (S′a) [EXACT for the frozen-window operator].** The cubic Dirac inverse controls the axis core (Proposition 9.58), and the exact six-component interval estimate controls the remaining annulus (Lemma 9.59). Endpoint observation and reconstruction retain the whole physical norm; no nonzero transport-scale limit is assumed. The lower bound tends to infinity, without a proved $4/7$ rate. Full-window numerics through $|au| = 10^{10}$ support growth near $|au|^{4/7}$ with no quasimode found. Details and proofs are in Attachment C and Attachment D. Remaining finite frequencies, surjectivity, the source-fold term and height outflow are separate obligations; global weak inboard patching comes after them.

Settled here, unconditionally.

- The **cone is circular with axis the radial shear**, computed in closed form (C1); pointwise realizability fails only on a measure-zero ray (C2); on-axis placement is exactly $\Theta > 0$ (C3).
- $a = 2 + \alpha_{\text{loc}}$, so in the exterior $v_s - 2 = \alpha$: **OA's cone-nondegeneracy condition is Paper I's window condition** (C4, §9A.3). Two derivations sharing no step, one inequality.
- The **moment condition is the stress return**, not a separate constraint (§9A.2).
- **Uniformity is structural**: compactness on the matching interval, and a *constant* margin α on the noncompact tail (§9A.5) — which discharges the concern Gondauri raised at §A.5.
- **The stress system is driven and linear**, so no invariant region exists; **radial state $\neq$ profile state** (§9C). Outcome G, proved.
- The **edge datum is the zero-stress defect** $(\Sigma_q, \Sigma_n/E)$, and the log factor cancels from the direction (§9C.3).
- Gondauri's one concrete algebraic flag is **discharged**: no defect (§2.3).

Settled conditionally. The moment map is a **submersion** — fibred over η, triangular on the midplane, cross term $O(\alpha\eta)$ — and a moment discrepancy μ is repairable without eating a margin δ_0 in width W exactly when $\mu \ll \delta_0 W^2$. Both are in Attachment A §§9.8–9.9; the second is a scaling identity, the first is

[COND] on the transcription of the source terms.

Errors found and corrected in this note. A conjecture that $\Theta > 0$ is *necessary* for a uniform margin — refuted by counterexample at $p_s = (0.5, 0.2)$ (§3.3). A coarse grid search that reported three feasible points as infeasible, the witnesses needing $\varepsilon \sim 10^{-7}$ (§3.1). And a first ending to this note that reduced the survivor to $\inf\Theta > 0$, which is on-axis only. Then, in §9D, three more of the same species: a cone feasibility programme reported *feasible* on a subsampled constraint grid while violating the cone between samples; the same programme reported *infeasible* because an absolute solver tolerance is vacuous where the row coefficients are 10^{-12}; and a conclusion that the perturbation must be supported outside the layer, which was an artifact of the second and is retracted. Three of the four are the same mistake — **a discretised inequality certifies nothing at a scale it does not resolve** — and it produces false confirmations as readily as false failures.

Obstruction tests, all failed. The scaling mechanism (both stress components are the same order); compact support forcing the direction out of a convex cone (the inequalities are homogeneous); moments versus a one-signed direction; a forced crossing of $M_2 = 0$ (nothing forces the trajectory); and cone degeneration at the top of the window (it degenerates at the bottom). **No obstruction to the architecture has been found.** One obstruction has been found to a class — the reflection-symmetric midplane cannot be completed inboard (Attachment A, Theorem 9.18) — and the non-symmetric midplane is closed (Attachment A, Theorems 9.25–9.26).

Surviving target: after Theorem 9.60: remaining finite frequencies, surjectivity, κt and outflow; then global weak inboard patching; $w(X_b, \eta) = 0$ from the layer.

The zero-stress base asked for by §9D is built in §9E, and at it the tangent perturbation is closed-form. Every cone condition is discharged on the midplane **layer** — pointwise (§3), tangentially (§9D.2), and constructively (§9E) — with both moments preserved identically. What is left is the preparation region inboard of the layer, where reflection symmetry fails (Attachment A, Theorem 9.18), and then the axial direction, which is where §9C said the missing resolution lived. The earlier formulation of this box, with moving endpoints $X_a(\eta), X_b(\eta)$, is withdrawn (Attachment A, §9.16.8).

An approximate solution with a margin, not an exact one — the correction being automatic by §§9.8–9.9 of Attachment A. That is what a construction supplies, which is why the existence claim of §8 is evidence of the right kind, **[COND-OA]**, pending independent review.